

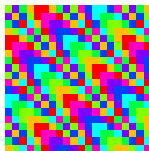
THÉORIE DES GRAPHS (5 BIS)

UNE APPLICATION DU PAGERANK

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Application du classement – Division 1 (2009–2010)

A bat B deux fois : $A \xleftarrow{6} B$

A bat B une fois et un nul : $A \xleftarrow{4} B$ et $A \xrightarrow{1} B$

A/B une victoire/une défaite : $A \xleftarrow{3} B$ et $A \xrightarrow{3} B$

A et B font deux nuls : $A \xleftarrow{2} B$ et $A \xrightarrow{2} B$

A perd une fois contre B et un nul : $A \xleftarrow{1} B$ et $A \xrightarrow{4} B$

A perd deux fois contre B : $A \xrightarrow{6} B$

$$\begin{pmatrix} * & 6 \\ 0 & * \end{pmatrix}, \begin{pmatrix} * & 4 \\ 1 & * \end{pmatrix}, \begin{pmatrix} * & 2 \\ 2 & * \end{pmatrix}, \begin{pmatrix} * & 3 \\ 3 & * \end{pmatrix}, \begin{pmatrix} * & 1 \\ 4 & * \end{pmatrix}, \begin{pmatrix} * & 0 \\ 6 & * \end{pmatrix}$$

Anderlecht, Cercle B., Club B., Charleroi, Courtrai, La Gantoise,
Genk, Beerschot, Lokeren, Malines, Roulers, Saint-Trond,
Standard, Westerlo, Zulte-W.

$$\begin{pmatrix} 0 & 0 & 3 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 6 & 1 & 0 & 0 \\ 6 & 0 & 6 & 0 & 4 & 6 & 3 & 3 & 1 & 1 & 3 & 1 & 1 & 3 & 4 \\ 3 & 0 & 0 & 0 & 1 & 1 & 4 & 3 & 0 & 4 & 0 & 1 & 3 & 0 & 1 \\ 6 & 6 & 6 & 0 & 4 & 6 & 3 & 1 & 3 & 6 & 0 & 2 & 4 & 4 & 2 \\ 6 & 1 & 4 & 1 & 0 & 1 & 1 & 3 & 1 & 1 & 0 & 3 & 6 & 2 & 3 \\ 2 & 0 & 4 & 0 & 4 & 0 & 1 & 4 & 0 & 0 & 0 & 3 & 0 & 4 & 6 \\ 6 & 3 & 1 & 3 & 4 & 4 & 0 & 4 & 0 & 6 & 2 & 1 & 3 & 1 & 2 \\ 6 & 3 & 3 & 4 & 3 & 1 & 1 & 0 & 3 & 6 & 1 & 3 & 2 & 1 & 4 \\ 6 & 4 & 6 & 3 & 4 & 6 & 6 & 3 & 0 & 3 & 3 & 6 & 6 & 3 & 4 \\ 6 & 4 & 1 & 0 & 4 & 6 & 0 & 0 & 3 & 0 & 0 & 6 & 4 & 3 & 3 \\ 6 & 3 & 6 & 6 & 6 & 6 & 2 & 4 & 3 & 6 & 0 & 6 & 3 & 2 & 1 \\ 0 & 4 & 4 & 2 & 3 & 3 & 4 & 3 & 0 & 0 & 0 & 0 & 1 & 6 & 6 \\ 4 & 4 & 3 & 1 & 0 & 6 & 3 & 2 & 0 & 1 & 3 & 4 & 0 & 3 & 2 \\ 6 & 3 & 6 & 1 & 2 & 1 & 4 & 4 & 3 & 3 & 2 & 0 & 3 & 0 & 6 \\ 6 & 1 & 4 & 2 & 3 & 0 & 2 & 1 & 1 & 3 & 4 & 0 & 2 & 0 & 0 \end{pmatrix}$$

$$H = S =$$

$$\begin{pmatrix} 0 & 0 & \frac{1}{4} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{12} & 0 & 0 \\ \frac{1}{7} & 0 & \frac{1}{7} & 0 & \frac{2}{21} & \frac{1}{7} & \frac{1}{14} & \frac{1}{14} & \frac{1}{42} & \frac{1}{14} & \frac{1}{42} & \frac{1}{14} & \frac{1}{42} & \frac{1}{14} & \frac{2}{21} \\ \frac{1}{7} & 0 & 0 & 0 & \frac{1}{21} & \frac{1}{21} & \frac{1}{4} & \frac{1}{14} & 0 & 0 & \frac{1}{21} & 0 & \frac{1}{42} & 0 & \frac{1}{21} \\ \frac{6}{53} & \frac{6}{53} & \frac{6}{53} & 0 & \frac{1}{21} & \frac{6}{53} & \frac{1}{53} & \frac{1}{53} & \frac{3}{53} & \frac{3}{53} & \frac{6}{53} & \frac{1}{53} & \frac{2}{53} & \frac{4}{53} & \frac{1}{53} \\ \frac{11}{2} & \frac{1}{33} & \frac{4}{33} & \frac{1}{33} & 0 & \frac{1}{33} & \frac{1}{33} & \frac{1}{11} & \frac{1}{33} & 0 & \frac{1}{11} & 0 & \frac{3}{28} & \frac{3}{33} & \frac{1}{11} \\ \frac{14}{3} & 0 & \frac{1}{7} & 0 & \frac{1}{7} & 0 & 0 & \frac{1}{7} & 0 & 0 & \frac{3}{28} & 0 & \frac{1}{28} & \frac{1}{7} & \frac{3}{14} \\ \frac{20}{6} & \frac{3}{40} & \frac{1}{40} & \frac{3}{40} & \frac{1}{10} & \frac{1}{40} & 0 & \frac{1}{10} & 0 & 0 & \frac{3}{20} & \frac{1}{20} & \frac{1}{40} & \frac{3}{40} & \frac{1}{20} \\ \frac{41}{2} & \frac{41}{3} & \frac{41}{4} & \frac{41}{4} & \frac{3}{41} & \frac{41}{4} & \frac{1}{41} & \frac{41}{2} & \frac{3}{41} & \frac{3}{41} & \frac{6}{41} & \frac{1}{41} & \frac{3}{41} & \frac{41}{4} & \frac{41}{4} \\ \frac{21}{3} & \frac{63}{4} & \frac{21}{2} & \frac{1}{21} & \frac{63}{4} & \frac{21}{4} & \frac{1}{21} & \frac{21}{3} & 0 & 0 & \frac{1}{21} & \frac{21}{2} & \frac{21}{3} & \frac{21}{4} & \frac{63}{3} \\ \frac{20}{1} & \frac{1}{10} & \frac{1}{40} & 0 & \frac{1}{63} & \frac{1}{10} & 0 & \frac{20}{1} & \frac{3}{40} & \frac{3}{40} & 0 & \frac{1}{20} & \frac{1}{20} & \frac{1}{40} & \frac{40}{3} \\ \frac{10}{0} & \frac{1}{20} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{30} & \frac{1}{10} & \frac{1}{20} & \frac{1}{20} & 0 & 0 & \frac{1}{10} & \frac{1}{30} & \frac{60}{1} \\ 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{18} & \frac{1}{12} & \frac{1}{10} & \frac{1}{12} & \frac{1}{10} & 0 & 0 & \frac{1}{40} & 0 & \frac{1}{10} & \frac{1}{30} & \frac{60}{1} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{18} & \frac{1}{12} & \frac{1}{10} & \frac{1}{9} & \frac{1}{12} & 0 & 0 & \frac{1}{40} & 0 & \frac{1}{10} & \frac{1}{30} & \frac{60}{1} \\ \frac{3}{22} & \frac{3}{44} & \frac{3}{4} & \frac{1}{36} & \frac{1}{12} & \frac{1}{10} & \frac{1}{12} & \frac{1}{18} & \frac{3}{41} & \frac{3}{41} & \frac{3}{36} & \frac{1}{12} & \frac{1}{36} & \frac{3}{12} & \frac{18}{3} \\ \frac{22}{6} & \frac{1}{1} & \frac{22}{4} & \frac{1}{44} & \frac{22}{3} & \frac{1}{2} & \frac{1}{44} & \frac{1}{11} & \frac{44}{29} & \frac{44}{29} & \frac{44}{3} & \frac{22}{4} & \frac{22}{4} & 0 & \frac{22}{0} \\ \frac{29}{29} & \frac{29}{29} & \frac{29}{29} & \frac{29}{29} & \frac{29}{29} & 0 & \frac{29}{29} & \frac{29}{29} & \frac{29}{29} & \frac{29}{29} & \frac{29}{29} & \frac{29}{29} & \frac{29}{29} & \frac{29}{29} & \frac{29}{29} \end{pmatrix}$$

$$G = 0.85H + 0.15J/15$$

.010	.010	.22	.010	.010	.15	.010	.010	.010	.010	.010	.43	.081	.010
.13	.010	.13	.010	.091	.13	.071	.071	.030	.030	.071	.030	.030	.071
.13	.010	.010	.010	.050	.050	.17	.13	.010	.17	.010	.050	.13	.010
.11	.11	.11	.010	.074	.11	.058	.026	.058	.11	.010	.042	.074	.074
.16	.036	.11	.036	.010	.036	.036	.087	.036	.036	.010	.087	.16	.062
.071	.010	.13	.010	.13	.010	.040	.13	.010	.010	.010	.10	.010	.13
.14	.074	.031	.074	.095	.095	.010	.095	.010	.14	.052	.031	.074	.031
.13	.072	.072	.093	.072	.031	.031	.010	.072	.13	.031	.072	.051	.031
.091	.064	.091	.050	.064	.091	.091	.050	.010	.050	.050	.091	.091	.050
.14	.095	.031	.010	.095	.14	.010	.010	.074	.010	.010	.14	.095	.074
.095	.052	.095	.095	.095	.095	.038	.067	.052	.095	.010	.095	.052	.038
.010	.10	.10	.057	.081	.081	.10	.081	.010	.010	.010	.010	.034	.15
.10	.10	.081	.034	.010	.15	.081	.057	.010	.034	.081	.10	.010	.081
.13	.068	.13	.029	.049	.029	.087	.087	.068	.068	.049	.010	.068	.010
.19	.039	.13	.069	.098	.010	.069	.039	.039	.098	.13	.010	.069	.010

Recherche d'un vecteur propre de valeur propre 1 (normalisé)

Anderlecht	0.103846
Cercle B.	0.0534133
Club B.	0.100507
Charleroi	0.0372727
Courtrai	0.0658507
La Gantoise	0.0788262
Genk	0.0632727
Beerschot	0.0661352
Lokeren	0.0298245
Malines	0.0647883
Roulers	0.0346681
Saint-Trond	0.09839
Standard	0.069503
Westerlo	0.0564951
Zulte	0.0772071

On ordonne :

	PageRank	classement	par points
0.103846	Anderlecht	Anderlecht	69
0.100507	Club B.	Club B.	57
0.09839	Saint-Trond	La Gantoise	49
0.0788262	La Gantoise	Courtrai	45
0.0772071	Zulte-W.	Saint-Trond	42
0.069503	Standard	Zulte-W.	41
0.0661352	Beerschot	Malines	39
0.0658507	Courtrai	Standard	39
0.0647883	Malines	Cercle B.	38
0.0632727	Genk	Beerschot	35
0.0564951	Westerlo	Genk	34
0.0534133	Cercle B.	Westerlo	32
0.0372727	Charleroi	Charleroi	23
0.0346681	Roulers	Lokeren	18
0.0298245	Lokeren	Roulers	18